Optimal Stopping for Drought Detection

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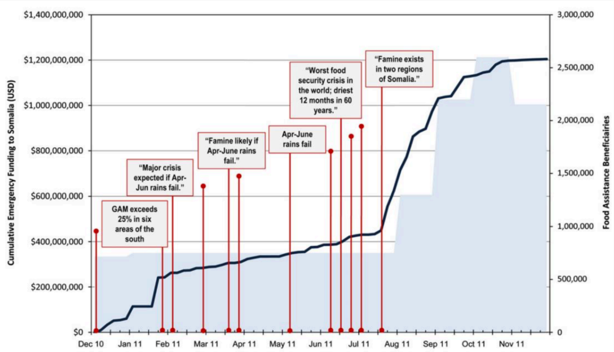
SAMBa ITT9

The Problem

- **Willis Towers Watson** arranges insurance against drought
- **Parametric insurance**: pay out based on proxy for damage
- **Goal**: find a trigger for when to send aid for drought



Optimal Stopping Problem

Problem is to find

$$\max_{\tau} \mathbb{E}\left[g(\tau, L_{\tau})\right]$$

- τ - stopping rule - when to send aid
- g - reward function:
  - benefit from action
  - constrain probability of false alarm
- L_t - likelihood of drought at time t

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Reward function g:

- Should incorporate:

	Yes extreme event	No extreme event
Yes forecast-based action	Correct prediction - worthy action	False alarm – act in vain
No forecast-based action	Miss – response cost	Correct rejection – no cost

Contingency table of possible outcomes

Source: Coughlan de Perez et al. 2015

- e.g. $g(t,l) = (f(t) - \gamma \mathbb{I}_{\{t<T\}})l$
- e.g. $g(t,l) = f(t)l^2 - \gamma \mathbb{I}_{\{t<T\}} l$

f decreasing in time, $\gamma > 0$ chosen to get correct probability of false alarm

T latest time drought can be declared

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Likelihood process L:

- Describes our belief at time t of how likely it is that a drought will happen, based on combining data sources
- $L_t \in [0, 1]$
- Stochastic process fitted to data
- For example:

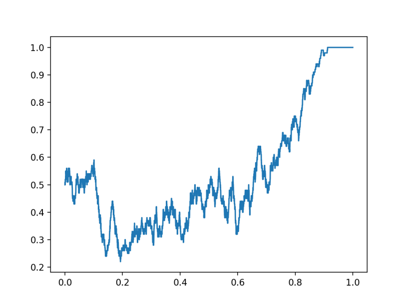
$$\mathrm{d}L_t = \sqrt{L_t(1 - L_t)}\,\mathrm{d}B_t$$

Wright-Fisher process

Simulation

Simulate the likelihood process $\mathrm{d}L_t = \sqrt{L_t(1 - L_t)}\,\mathrm{d}B_t$:

$$L_{i+1} = \begin{cases} L_i + \Delta x & \text{with prob.} \quad \frac{p}{2}, \\ L_i - \Delta x & \text{with prob.} \quad \frac{p}{2}, \\ L_i & \text{with prob.} \quad 1 - p. \end{cases}$$



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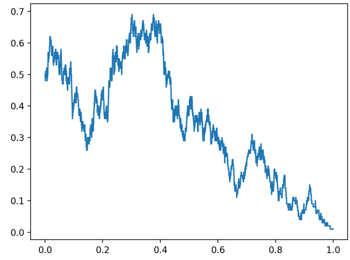

Stopping Boundary

Problem is to find

$$\max_{\tau} \mathbb{E}\left[g(\tau, L_\tau)\right]$$

Optimal stopping rule $\tau^\star$ is first time to hit a boundary.

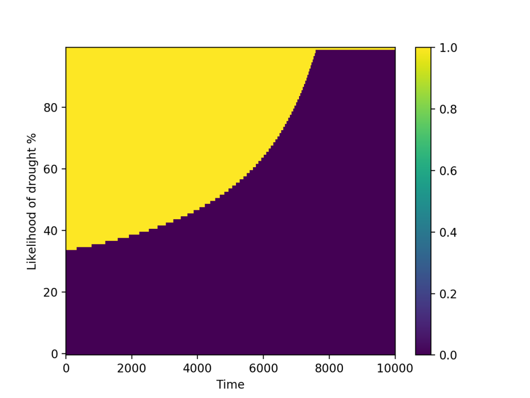

Figure 1: $g(t, l) = f(t)l^2 - \gamma \mathbb{I}_{\{t<T\}} l$

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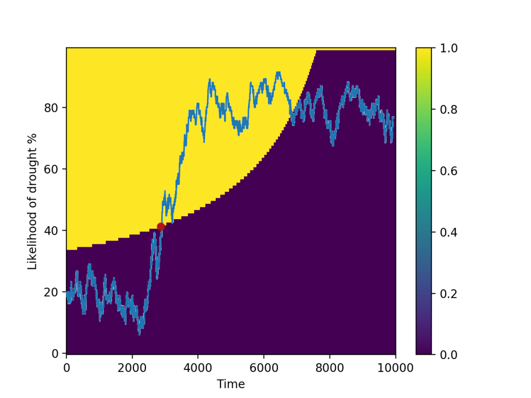

Figure 2: $g(t, l) = f(t)l^2 - \gamma \mathbb{I}_{\{t<T\}} l$

Free Boundary Problem

Problem is to find

$$v(t, l) := \max_{\tau} \mathbb{E}^{(t,l)} \left[g(\tau, L_\tau) \right]$$

Value function v and boundary function b expected to solve free boundary problem:

$$\begin{cases} \frac{\partial v}{\partial t}(t, l) = -\frac{1}{2} l(1-l) \frac{\partial^2 v}{\partial l^2}(t, l), & l < b(t), \\ v(t, l) = g(t, l), & l = b(t), \\ \frac{\partial v}{\partial t}(t, b(t)-) = \frac{\partial g}{\partial t}(t, b(t)). \end{cases}$$

Future Work

- Investigate more suitable reward functions
- Prove that the stopping boundary is increasing in time
- Analysis of free boundary problem
- Find a simple stopping rule that is a good approximation
- Incorporate multiple actions - sequential optimal stopping
- Fit underlying process to data and validate model